THE MEASUREMENT OF HIGH VOLTAGE OF ALTERNATING CURRENT BY KERR CELL.

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Introduction.

Many experiments have been carried out on the double refraction of nitrobenzene in the electric field by several authors, and its Kerr constant was evaluated by different methods. The present authors tried to evaluate the voltage applied to the electric field by using the known Kerr constant and the intensity of light which was passed through the Kerr cell of nitrobenzene placed in crossed nicols and simultaneously in the field of 7,000–25,000 volt/cm. and of 50-cycle of alternating current.

Experimental.

Apparatus. The glass vessel which holds nitrobenzene has a capacity of 8 cm.×2 cm.×3.5 cm. In order to pass the light through the vessel a hole of 6 mm. of diameter is made on each wall of 2 cm.×3.5 cm. and it is covered with a sheet of deck glass which is fixed at the wall with bakerite varnish by gently heating after dried. Two copper plates of 0.3 mm. of thickness and 71.0 mm.×6.0 mm. of area are put parallel with each other with 0.38 cm. of width and these two are fixed perpendicularly on the thick glass cover of the vessel and are used as electrodes. It is necessary to use some sealing materials to make the cover tight with the vessel in order to prevent the absorption of moisture into nitrobenzene. Such materials as vaseline, however, are not appropriate, as they are soluble in nitrobenzene. Therefore the above glass vessel is put into a larger glass vessel (9 cm.×3.5 cm.×6.0 cm.) having as well windows to admit light as before. Phosphorus pentoxide is put on the bottom of the outer vessel to absorb moisture in the vessels.

A gas-filled tungsten filament lamp of 100-volt and 100-watt is illuminated by secondary batteries as the light source. The light is condenced by a lens and is made approximately a monochromatic light by the filter of 510–540 $\mu\mu$. The light passed through the slits and a polarizer of nicol is admitted into nitrobenzene in the glass vessel. The double refracted light in nitrobenzene is passed through an analyser and is fallen on a dry plate in a special camera, and a spot of 2.0 mm. of diameter is made on the plate. The dry plate is shifted a little after each experiment to obtain a series of spots on the plate. The dry plates used were Ilford screened chromatic H.D. 270.

The time of exposure of the dry plate was 5–10 sec. The accuracy to 0.05 sec. of time of exposure was obtained by using the time of flow of a certain quantity of an aqueous solution of glycerine through a capillary tube. The angle of rotation of the analyser was read with a magnifying lens.

Electric high potentials were obtained from the transformer with the turn ratios of 30, 50, 75 and 100. Alternating current of 110-volt and 50-cycle was supplied to the primary circuit of the transformer and it was controlled to 100 volts.

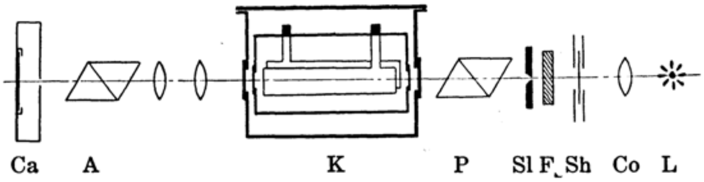

Fig. 1.

The apparatus used is illustrated in Fig. 1. L is the light source, Co condenser, Sh shutter, F filter, Sl slit, P polarizer, K Kerr cell, A analyser and Ca camera.

Nitrobenzene used was the pure chemicals of Kahlbaum, and it was purified as follows. Nitrobenzene (500 gr.) was distilled at 130°C. under a reduced pressure by passing dry air from the capillary. The distillate was frozen by freezing mixture and the part which did not melt at 1.0°C. was collected. This part was put into a distilling apparatus whole of which consisted of glass, and it was distilled without boiling under 2 mm. pressure by heating one part of the vessel below 90°C. and by cooling the other part with freezing mixture. About 100 c.c. of thus purified nitrobenzene was obtained and kept free from atmosphere in the dark.

Experimental Procedure. Nitrobenzene is sealed in the cleaned vessel. The position of the polarizer is so adjusted as its plane of polarization makes the angle of 45° to the plane of electrode. The camera is unmounted and the analyser is rotated until the visual field is just dark to the naked eye. The camera in which the halved dry plate is placed is mounted on the apparatus and a high potential is applied on the electrodes. A requisite time of exposure is given to the dry plate by opening the shutter. The dry plate is shifted a little and another exposure is carried out as before by changing the potential. A series of points of exposures is thus obtained on a sheet of dry plate.

The darkness of the spots of the dry plate is not proportional to the intensity of light fallen on it. Therefore the spots of standards for the comparison of darkness with intensity of light were made on the dry plate of the other half by changing the angles of polarization between the polarizer and the analyser to 90°, 80°, 70°, 60°, 50°, 40°, 30°, 20° and 10° with the equal time of exposure and with no potential on the electrodes of the Kerr cell. The one half of the dry plate used for determining the potentials and the other half of the plate used for obtaining the standards of darkness were developed and fixed together under the same condition. The darkness of the plates was measured with a microphotometer. And the intensity of light fallen on the former plate was determined by comparing with the standard prepared on the latter.

Every one series of experiment was carried out by changing the turn ratios of transformer to 30, 50, 75 and 100, and the standard of darkness was prepared in every one series. The voltage of the primary circuit was controlled to 100 volts during the experiments, but the pointer of voltmeter fluctuated between 98 and 102 volts. Although the temperature varied 10–15°C. in the experiments, its effect, however, was not taken into the calculation.

Intensity of Light. When a potential is not applied to the Kerr cell, if α denote the angle of polarization between the polarizer and analyser, and I_α denote the intensity of light from the analyser, and $I = 1$ when $\alpha = 0°$, then there is the following relation.

$$I_\alpha = \cos^2\alpha \, .$$

The darkness of the standard dry plate is read with the microphotometer and a curve relating between the reading and I_α is drawn. The intensity of light fallen on the dry plate used for the determination of potential is determined with the reading of the microphotometer by using the above curve. If the product of mean intensity of light and time of exposure, i.e. light quantity is equal each other, it is assumed that the darkness of the dry plate is the same, though the mode of change of intensity of light may be different in the course of exposure.

Relation between Voltage and Intensity of Light. Since an alternating current of 50-cycle is used and the time of exposure of the dry plate is 5–10 sec., the intensity of light varies periodically in the course of exposing and the mean of intensity is recorded on the dry plate. Hence the effective voltage of A.C. is calculated from the measured mean intensity of light.

When the Kerr cell is put in the crossed nicols and the angle between the plane of polarization and that of electrode is 45°, then the relation between the intensity of light I from analyser and voltage E is as follows.

$$I = \sin^2 \frac{\pi B l E^2}{a^2}$$

where E is the voltage expressed in e.s.u., B the Kerr constant, a the distance between electrodes and l the width of electric field.

If E_t denote the voltage at the time t since the voltage E of A.C. changes with time, I_t the intensity of light at that time, and also ϵ_t the effective voltage when the voltage of A.C. changes in the form of a sine curve, then

$$I_t = \sin^2\left(\frac{2\pi B l \epsilon^2}{a^2} \sin^2 \omega t\right)$$

If I denote the mean intensity of light with ¼ cycle,

$$\begin{aligned} I &= \frac{2\omega}{\pi} \sum_{t=0}^{t=\frac{\pi}{2\omega}} I_t \\ &= \frac{2\omega}{\pi} \int_0^{\frac{\pi}{2\omega}} \sin^2\left(\frac{2\pi B l \epsilon^2}{a^2} \sin^2 \omega t\right) dt \,. \end{aligned}$$

When $2\pi B l \epsilon^2 / a^2$ is represented by b,

$$I = \frac{2\omega}{\pi} \int_0^{\frac{\pi}{2\omega}} \sin^2 (b \sin^2 \omega t)\, dt$$

$$\begin{aligned} I &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left\{1 - \cos (2b \sin^2 x)\right\} dx \\ &= \frac{1}{2} - \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \Big[\cos b \cos (b \cos 2x) - \sin b \sin (b \cos 2x)\Big]\, dx \\ &= \frac{1}{2} - \frac{2}{\pi} \cos b \int_0^{\frac{\pi}{4}} \cos (b \cos x)\, dx \\ &= \frac{1}{2} \left\{1 - \cos\left(\frac{2\pi B l \epsilon^2}{a^2}\right) J_0\left(\frac{2\pi B l \epsilon^2}{a^2}\right)\right\} \end{aligned}$$

where J_0 indicates Bessel's function.

The value of B at 520 $\mu\mu$ of λ is obtained by the following equation as 374.4×10^{-7} from the value of B $(= 346.0 \times 10^{-7})$ at 546 $\mu\mu$ of λ which was obtained by R. Möller.⁽¹⁾

$$B_{\lambda_1} = B_{\lambda_2} \frac{\lambda_2 n_{\lambda_2}(n_{\lambda_1}-1)^2}{\lambda_1 n_{\lambda_1}(n_{\lambda_2}-1)^2}$$

where n is the index of refraction.

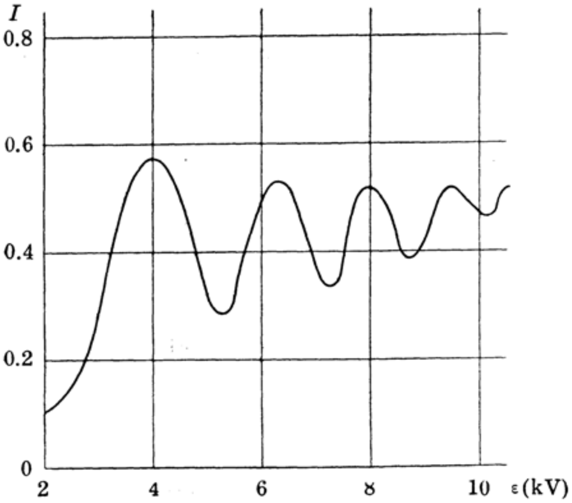

Fig. 2.

The constants of the apparatus are as follows: $a = 0.38$ cm., the length of electrode is 7.10 cm., hence $l = 7.18$ cm. (by calculation). Putting these values in the above equation,

$$I = \frac{1}{2}\left\{1 - \cos(7.45^\circ \epsilon^2) J_0(0.1300\,\epsilon^2)\right\}$$

where ϵ is expressed in kilovolt. The relation between ϵ and I in the range from $\epsilon = 2$ to 10.5 is shown by a curve in Fig. 2.

The Results of Experiment. The effective voltages ϵ'_2 corresponding to the secondary voltages ϵ_2 obtained from the turn ratios of the transformer were calculated from the measured intensity of light by using the curve of Fig. 2. These values are shown in Table 1. ϵ_1 in the table is the effective voltage of the primary circuit, I intensity of light, ϵ_2 effective voltage of the secondary obtained by the turn ratio and ϵ'_2 that obtained from the intensity of light.

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(1) *Phys. Z.*, **30** (1929), 22.

Table 1.

ε_1	I	ε_2	ε'_2	$\varepsilon_2/\varepsilon_1$	$\varepsilon'_2/\varepsilon_1$
100.0	0.22	3.000	2.780	30	27.8
,,	0.27	,,	2.920	,,	29.2
,,	0.24	,,	2.850	,,	28.5
,,	0.22	,,	2.780	,,	27.8
,,	0.255	,,	2.900	,,	29.0
80.5	0.21	2.415	2.760	,,	33.0
90.0	0.32	2.700	3.050	,,	33.9
100.0	0.45	5.000	4.620	50	46.2
,,	0.395	,,	4.760	,,	47.6
,,	0.395	,,	4.760	,,	47.6
70.0	0.33	3.500	3.070	,,	49.5
100.0	0.41	7.500	7.510	75	75.1
,,	0.43	,,	7.550	,,	75.5
,,	0.40	,,	7.500	,,	75.0
,,	0.40	,,	7.500	,,	75.0
,,	0.34	,,	7.300	,,	73.0
,,	0.42	,,	7.520	,,	75.2
100	0.43	10.000	9.020	100	90.2
,,	0.40	,,	8.900	,,	89.0
,,	0.48	,,	9.180	,,	91.8
71.0	0.52	7.100	6.450	,,	90.9
76.0	0.435	7.600	6.780	,,	89.3

Discussion of Results. The voltages obtained from the measured intensity of light are nearly equal to the voltage calculated from the turn ratios of transformer, but the former is generally less than the latter. Every value of voltages obtained from the measured intensity of light has a fluctuation of about 3% from their mean. The following few facts may be the main causes of the above fluctuation.

(1) The primary voltage always varied in the range from 1 to 4% in the measurements. (2) The accuracy of the microphotometer used is 3–5%. (3) Light passed through the filter is 510–540 $\mu\mu$, while it is assumed simply 520 $\mu\mu$. (4) There is the error of less than 1% in the value of Kerr constant. According to Hehlgans(2) its value considerably

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(2) *Phys. Z.*, **30** (1929), 942.

varies with the purity of nitrobenzene. (5) Potential gradient of electric field is not linear in impure nitrobenzene.

Summary.

(1) The monochromatic light of about ($\lambda =$) 520 $\mu\mu$ was passed through nitrobenzene which was put in crossed nicols and in an alternating electric field of 7,000–25,000 volts/cm., and the intensity of light from the analyser was measured.

(2) The potential of the electric field was calculated from the measured intensity of light by using the known Kerr constant.

(3) The calculated values of potentials are nearly equal to those obtained from the turn ratios of the transformer, but are generally less than the latter.

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